ENGLISH VERSION..

Análisis sísmico comparativo de una estructura irregular torsionalmente flexible aplicando NSP, MPA, NLRHA

Comparative seismic analysis of a torsionally-flexible unsymmetric structure by applying NSP, MPA, NLRHA

C. Medina[1]*, D. Galarza*

* Universidad Técnica de Ambato – Ambato, ECUADOR

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Abstract

The applicability of nonlinear analysis methods: pushover (NSP) and multimodal pushover (MPA) is verified for torsionally – flexible structures whose natural vibration period is larger than 1sec. The results are confronted with a nonlinear response history analysis (NLRHA). The procedure is applied to an irregular 10-story concrete framed structure, built in the 70's. A mathematical three-dimensional finite element-based model was developed using material properties based on Mander (1984) concrete model and Park (1975) steel model. Seismic hazard is established for the design basis earthquake with 10% probability of exceedance in 50 years; and, for 3 pair of ground motions properly matched and scaled to DBE. It was determined that for structures with 1sec or more natural vibration period, NSP underestimates the maximum displacement capacity, since it does not consider the contribution of higher vibration modes to the total response of the system. MPA fits more to NLRHA and is mainly applicable to regular framed buildings, since inconsistences are generated when a considerable percentage of torsion occurs in translational vibration modes.

Keywords: *Seismic performance; nonlinear analysis; NSP; MPA; NLRHA*

Resumen

Se verifica la aplicabilidad de los métodos de análisis no lineal: pushover (NSP) y pushover multimodal (MPA) para estructuras torsionalmente flexibles cuyo período de vibración fundamental es mayor a 1s, confrontando los resultados con un análisis de historia de respuesta no lineal (NLRHA). El análisis se aplica a un edificio irregular en planta y elevación de 10 pisos porticado en concreto reforzado construido en los años 70's. Se construyó un modelo matemático tridimensional basado en elementos finitos, utilizando propiedades no lineales de los materiales en base a los modelos constitutivos de Mander (1988) y Park (1975) para el concreto y acero respectivamente. La demanda se estableció para el sismo de diseño definido por NEC-SE-DS2015 cuya probabilidad de excedencia es del 10% en 50 años; y para 3 pares de registros sísmicos propiamente seleccionados, ajustados y escalados al espectro de diseño. Se determinó que para estructuras de más de 1s de período de vibración el NSP subestima la capacidad de desplazamiento máximo, puesto que no considera el aporte de los modos de vibración superiores a la respuesta total del sistema. El MPA se ajusta más al NLRHA y es aplicable principalmente en estructuras regulares, pues se generan inconsistencias cuando se presenta un porcentaje de torsión considerable en modos de vibración traslacionales.

Palabras clave: Desempeño sísmico; análisis no lineal; NSP; MPA; NLRHA

1. Introduction

The reinforced concrete structures built decades ago are vulnerable because they respond to the level of knowledge, computational progress and quality of the materials of that time. These buildings exist in large cities, so it is urgent to verify the performance of these structures against seismic events (Aguiar, 2016) (Mouzzoun et al., 2013).

For the evaluation of building structures, several inelastic analysis procedures are available, with weaknesses and strengths. Their selection depends mainly on the accepted level of uncertainty and resources available (ATC-40, 1996)*. The methods of analysis include nonlinear static pushover analysis, multimodal pushover analysis and nonlinear response history analysis.*

The pushover method (NSP) is an efficient procedure to evaluate the capacity of a structure. It consists in simplifying it in a mathematical model of a degree of freedom, subjected to the monotonic incremental application of lateral loads that represent the inertial forces during an earthquake, until the system collapses (FEMA 440, 2005).

The analysis is based on the fact that the complex dynamic response is governed by the fundamental vibration period, which restricts its scope to structures whose percentage of modal mass excited in the 1st mode reaches 75%. This often occurs when the fundamental period is less than 1 sec (Handana et al., 2018).

[1] ***Corresponding author:***
Universidad Técnica de Ambato – Ambato, ECUADOR
E-mail: cd.medina@uta.edu.ec

For this reason, since the behavior of these structures under seismic loads cannot be described only in the first vibration mode, a multimodal pushover analysis (MPA) could be considered. This analysis considers the contribution of superior vibration modes in the dynamic response, especially in structures with a natural vibration period of more than 1 second (Campbell et al., 2010) (Campbell , 2008).

The MPA is based on the fact that the maximum response of the elastic structure due to its n vibration mode can be accurately estimated by conducting a pushover analysis of the structure, subject to lateral loads distributed over the height of the building according to the modal shape of the n mode $\emptyset_n$.

$$\boldsymbol{s_n = m * \emptyset_n} \tag{1}$$

The structure is pushed to a displacement on the roof u_{rno} *determined from the peak deformation of the n mode of the elastic system with 1 degree of freedom.*

$$\boldsymbol{u_{rno} = \Gamma_n * \emptyset_{rn} * D_n} \tag{2}$$

Then, using a modal combination process (e.g., SRSS, CQC), the maximum responses of each vibration mode are combined, leading to the MPA procedure (Chopra y Goel, 2001).

This methodology is widely applied to structures whose higher vibration modes significantly influence the dynamic response. However, the procedure is limited to torsionally- rigid structures with the first two modes of vibration predominantly translational since the static response of the structure is qualitatively similar to the dynamic behavior. Therefore, only translational modes are considered; and if a significant percentage of torsion appears in the translational modes, the response might be distorted (Chopra y Goel, 2003).

For this reason, the nonlinear response history analysis (NLRHA) is the alternative that presents better results to seismic demands. Due to its complexity and high standards, it goes beyond practical application and is appropriate for research and analysis of special importance structures.

The method consists of a sophisticated approach to examine the inelastic demands produced on a structure by a set of ground acceleration histories (ASCE/SEI 41-17, 2017). *The force-deformation relationship of each structural element subject to cyclic deformation must now be adjusted to a hysteretic model. The initial load curve is nonlinear at larger deformation rates, and the discharge curves differ from the initial load branch. This defines the local behavior of the elements, which, when analyzed together, result in the global response history of the structure for each time period (Chopra, 2016).*

In this sense, the objective of the research is to carry out a comparative seismic analysis of a torsionally-flexible structure with a vibration period greater than 1 second to verify the applicability of the nonlinear pushover analysis (NSP) and multimodal pushover analysis (MPA) methods in comparison with the nonlinear response history analysis (NLRHA).

2. Methodology

The comparative analysis was applied to a 10-story reinforced concrete porticoed building with a basement. It is a type 2 unsymmetric-plan building with excessive inward corners, making it susceptible to torsional flexibility, and it has a type 3 irregularity in elevation with vertical geometric irregularity.

To represent the building, a three-dimensional mathematical model based on finite elements is used. Its materials simulate nonlinear behavior through their constitutive models. The concentrated plasticity model based on a fiber model was used to monitor the behavior of the structural elements at the sites where nonlinearity is expected. (Figure 1) shows the model used.

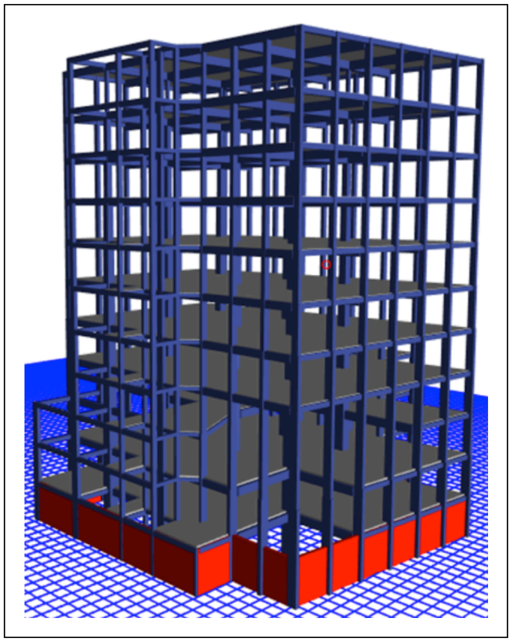

Figure 1. *Mathematical model used*

2.1 Constituent Materials: Concrete

The on-site concrete resistance verified through a non-destructive test is 28 MPa. The constitutive model used is the one proposed by (Mander et al., 1988), This model considers the additional strength and ductility that the confinement reinforcement provides to the concrete core. For the immediate occupation level, a deformation of 0.003 was set in terms of acceptance criteria, corresponding to the limit before the detachment of the unconfined cover. A value close to the deformation was set for the life safety level, corresponding to the maximum stress of confined concrete. And for the collapse prevention level, the life safety deformation was limited to 2 times before the fracture of the confined concrete core (Priestley et al., 2007). (Figure 2) shows the model used.

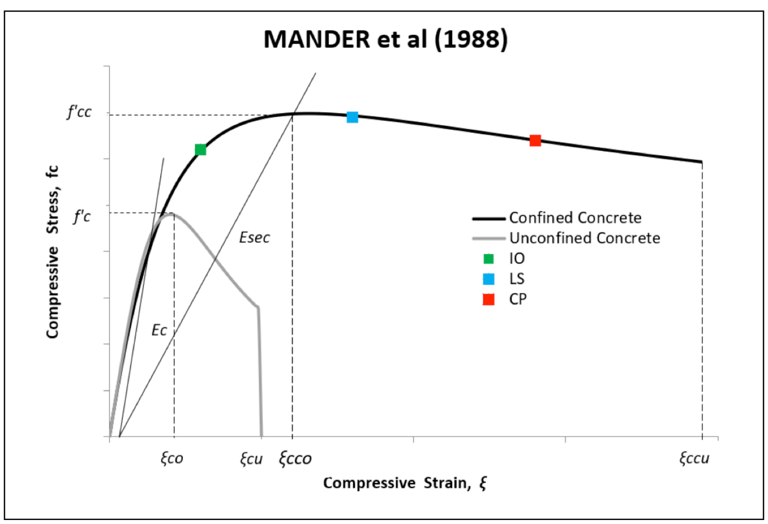

Figure 2. Constitutive model for concrete

The nonlinear dynamic analysis considers the (Takeda, 1970) model shown in (Figure 3), which is appropriate for brittle materials since it uses a degraded hysteretic curve in the discharge along the elastic segments. When loading again, the curve follows a secant line to the loading curve in the opposite direction. The target point of the secant line is at the maximum deformation that occurs in that direction under the previous load cycles. This results in a decreasing amount of energy dissipation with large deformations (Takeda et al., 1970).

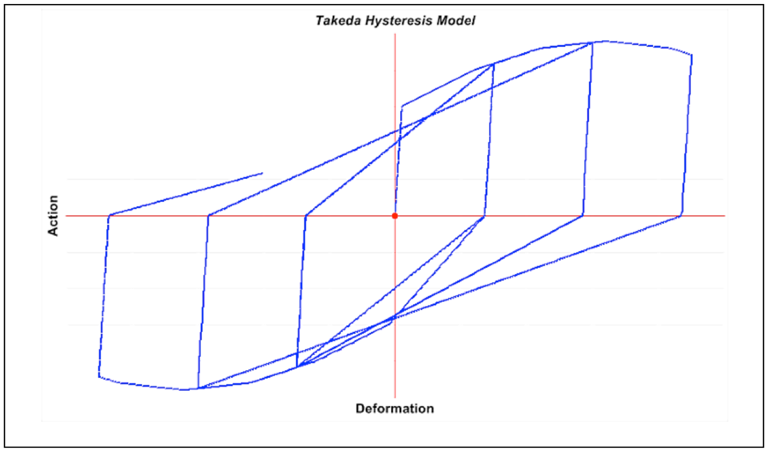

Figure 3. *Hysteretic model of concrete*

2.2 Constituent Materials: Steel

The creep strength value of the transverse and longitudinal steel bars of the building is 280 MPa due to construction time. The behavior model of (Park and Paulay, 1975) was used. This model considers the post creep hardening of concrete with a parabolic trend until rupture, shown in (Figure 4). The deformation for immediate occupation was limited to 0.010, corresponding to the beginning of the post creep hardening. The life safety level was limited to 0.020, corresponding to the beginning of the possible buckling in the longitudinal bars, and for collapse prevention, a deformation equivalent to 60% rupture strain was set.

In the dynamic nonlinear analysis NLRHA, a kinematic hardening hysteretic model was used, which is appropriate for ductile materials that allow dissipating large amounts of energy, which, in the loading and unloading process, the curve follows a path made of parallel segments and with the same length as the previously loaded segments. (Figure 5) shows the model.

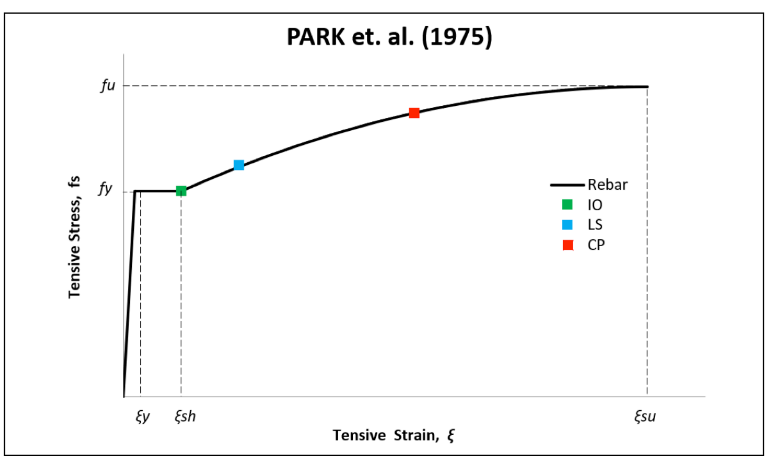

Figure 4. *Constitutive model of steel*

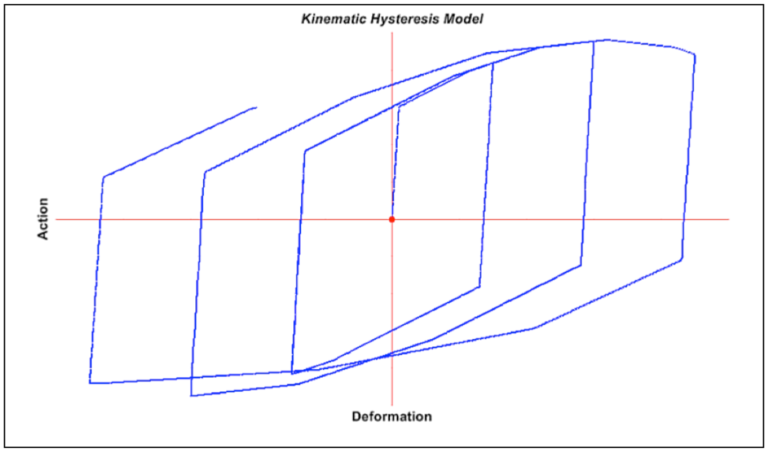

Figure 5. *Hysteretic model of steel*

2.3 Constituent models: H.A. Sections

The nonlinearity of the sections was represented through a concentrated plasticity model based on uniaxial fibers distributed in the cross-section. Its constituents correspond to the previously defined deformation stress curves and hysteretic models. In general, three types of fibers are used: for confined concrete, for unconfined concrete and for steel. The mesh consists of nine fibers representing the section cover, nine fibers to represent the confined concrete core and one fiber for each steel bar. (Figure 6) shows an example of the one-column section model used in the analyses.

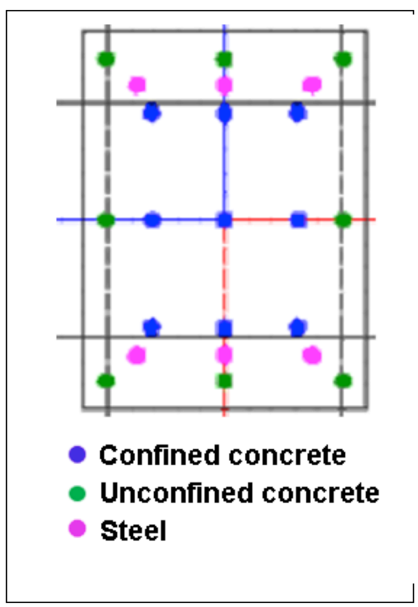

Figure 6. *Cross-section fiber model*

2.4 Nonlinear Static Pushover Analysis NSP

In the model, only the primary structural elements contributing to the structural system resistant to lateral load have been considered, so stairs and elevators were discarded.

The pattern to be adopted depends on the dynamic characteristics of the structure. Thus, when the percentage of modal participation of the mass associated with the fundamental mode exceeds 75% in the analyzed direction, the load can be distributed according to the values of floor shear obtained through the static analysis; or, otherwise, the load can be distributed according to the modal form associated to the fundamental vibration mode in the analyzed direction.

In this case, the first mode could not accumulate 75% of the modal mass in any direction, so the pattern adopted is based on the modal shape.

The seismic demand is estimated for the designed earthquake according to (NEC-SE-DS, 2015), for a probability of exceedance of 10% in 50 years with a return period of 475 years. The structure is located on a C medium stiffness soil, in a high seismic hazard zone $PGA_{ROCK} = 0.40g$.

2.5 Multimodal Pushover Analysis MPA

The modal analysis of the structure was carried out to obtain the vibration modes and modal mass participation percentages. Once analyzed, they were grouped according to the predominant translation direction.

The spatial distribution of the effective seismic forces$[M]\{l\}_i$ *can be expanded as a sum of the modal inertial force distribution as follows:*

$$[\boldsymbol{M}]\{\boldsymbol{l}\}_{\boldsymbol{i}} = \sum_{n=1}^{3N} \{\boldsymbol{s}_{\boldsymbol{n}}^{\boldsymbol{i}}\} = \sum_{n=1}^{3N} \boldsymbol{\Gamma}_{\boldsymbol{ni}}\,[\boldsymbol{M}]\{\boldsymbol{\phi}_{\boldsymbol{n}}\} \qquad (3)$$

Four vibration modes were taken for each orthogonal direction to excite the greatest amount of modal mass. Thus, for the analyzed direction X, vibration modes 2, 5, 8, 11 were grouped together, while for the direction Y, modes 1, 4, 7, 10 were grouped together.

(Table 1) shows that, although translation predominates in these modes, a certain percentage of torsion is present, which is characteristic of a torsionally-flexible structure. Direction Y shows a higher percentage of excited mass in rotation.

Table 1. *Modal mass participation factors*

Mode	Period	ux	uy	rz	Direction
1	1.66	0.003	0.44	0.241	Y
2	1.56	0.592	0.014	0.07	X
3	1.5	0.073	0.214	0.34	Torsional
4	0.6	0.012	0.062	0.017	Y
5	0.57	0.082	0.01	0	X
6	0.55	0	0.021	0.073	Torsional
7	0.36	0.009	0.02	0.004	Y
8	0.34	0.022	0.011	0.002	X
9	0.32	0.002	0.003	0.024	Torsional
10	0.241	0.0072	0.0109	0.0031	Y
11	0.232	0.0105	0.0094	0.0019	X
12	0.222	0.0032	0.0008	0.0136	Torsional

(Figure 7) and (Figure 8) show the modal forms of the structure for each orthogonal direction used for the MPA.

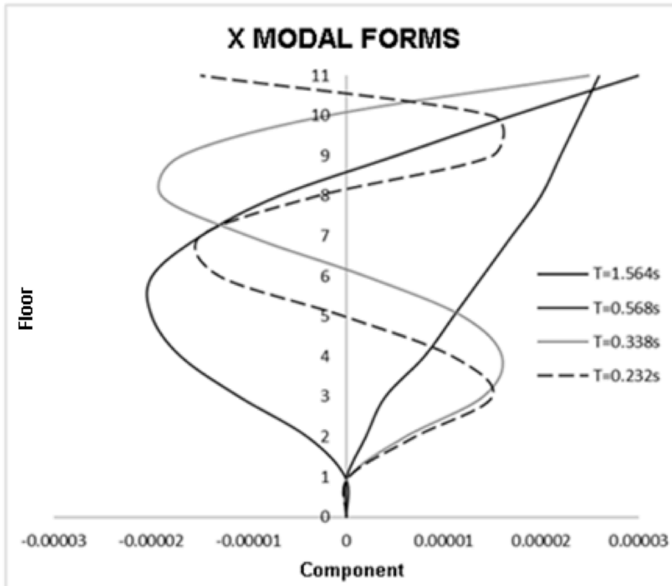

Figure 7. Modal forms of direction X

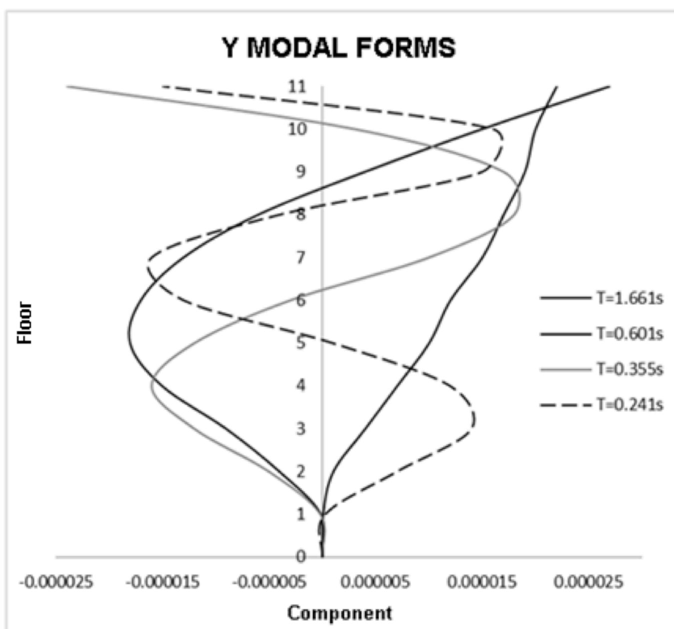

Figure 8. Modal forms of direction Y

With these force distributions, the pushover analysis is performed for each vibration mode. It is important to mention that the 3D pushover analysis is limited to torsionally-rigid buildings in which the first two vibration periods are predominantly translational. The static response is qualitatively similar to the dynamic behavior.

(Table 1) shows that there is a certain percentage of torsion in the first two vibration modes. For this reason, one of the objectives of the study is to determine how reliable the MPA results are for torsionally-flexible structures.

The modal combination used to obtain the total response corresponds to the root of the sum of the squares of the shape:

$$\boldsymbol{r_o} \simeq \left(\sum_{n=1}^{N} \boldsymbol{r_{no}^2}\right)^{\frac{1}{2}} \qquad \textbf{(4)}$$

This rule provides excellent response estimates for structures with widely separated natural frequencies (Chopra, 2016).

2.6 Nonlinear response history analysis NLRHA

Three pairs of seismic records were used to determine the displacement demand, from which the maximum response was taken (ASCE/SEI 41-17, 2017). The selection was made based on the tectonic characteristics of the implementation site of the structure, which is settled on a high-density rigid ground in a regime of superficial inverse faults. The Ambato earthquake (1949/8/5) was taken as reference, Mw = 6.8, and depth < 15km (Instituto Geofísico: Escuela Politécnica Nacional, 2013).

(Table 2) and (Figure 9) show the records that meet the indicated conditions.

Table 2. *Selected acceleration records*

Info / Event	Northridge	San Fernando	Loma Prieta
Date	1994/01/17	1971/02/09	1989/10/17
Station	Sylmar	Sylmar	Corralitos
Soil type	C	C	C
Distance(km)	19.2	7.3	7.1
Mw	6.4	6.6	7.0
PGA (g)	1.53	1.251	0.64
Fault mechanism	Crustal	Crustal	Crustal
Depth (km)	19	9	18

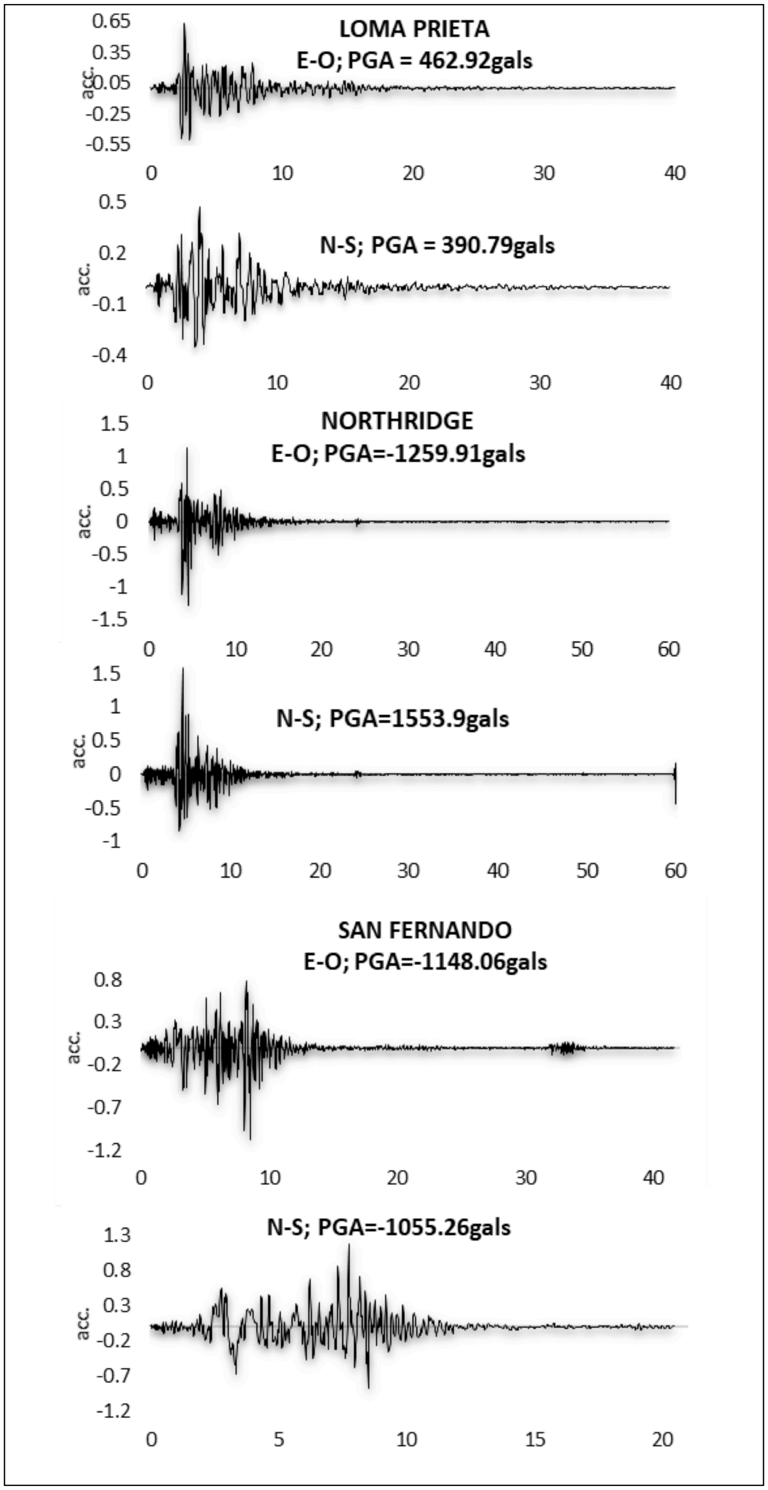

Figure 9. *Selected acceleration records*

2.7 Spectral Adjustment

An adjustment was made to the target spectrum in the time domain as this implies greater precision in obtaining results. This method adjusts the acceleration stories in the time domain by adding wavelets. A wavelet is a mathematical function that defines a waveform of limited effective duration with a percentage of zero. It usually starts at zero, grows and decreases again to zero. While the spectral adjustment procedure in the time domain is

generally more complicated than the approximation in the frequency domain, it has good convergence and, in most cases, retains the non-stationary character of the reference time series (Al Atik and Abrahamson, 2010).
The records were scaled so that the average value of the spectra coming from the square root of the sum of the squares of the spectra in the records is not below the target spectrum for periods between 0.2T and 1.5T (ASCE/SEI 41-17, 2017), In the case under study, this corresponds to 0.33sec - 2.5sec for direction X and 0.31sec - 2.35sec for direction Y.

The adjustment and scaling process consists of two parts. First, the adjustment was made in the aforementioned time domain. However, since certain spectral ordinates of the average SRSS were still below the target spectrum, additional scaling values were applied, as shown in (Table 3).

Table 3. *Scale factors for seismic records*

Event	Component	Scale Factor
Northridge	E-O	1.13
	N-S	1.13
San Fernando	E-O	1.14
	N-S	1.14
Loma Prieta	E-O	1.12
	N-S	1.12

(Figure 10) shows the target spectrum and the average root spectrum of the sum of the SRSS squares of the record pairs as a result of the spectral scaling and adjustment process.

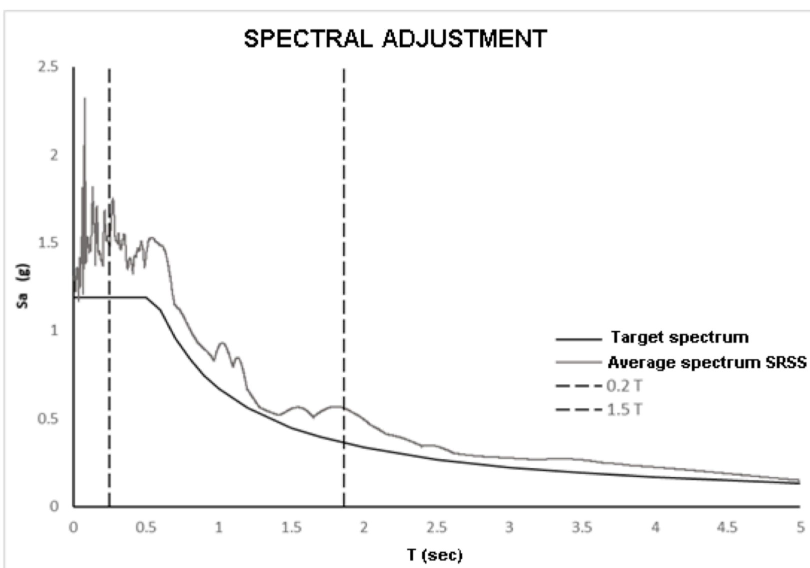

Figure 10. *Adjustment and spectral scaling*

3. Results Y Discussion

3.1 Pushover analysis

(Figure 11) and (Figure 12) show the capacity curves of the structure for the horizontal directions X and Y, respectively. A comparison is made between the NSP curve and the MPA curves for each vibration mode.

It is observed that the load pattern considered in the pushover analysis greatly influences the rigidity with which the structure responds. Thus, the higher the vibration mode, the greater the rigidity of the structure, but its displacement capacity is lower.

Additionally, it is shown that the capacity curve obtained with the load pattern for the 1st vibration mode is very similar to the one obtained with the load pattern from the equivalent lateral force method.

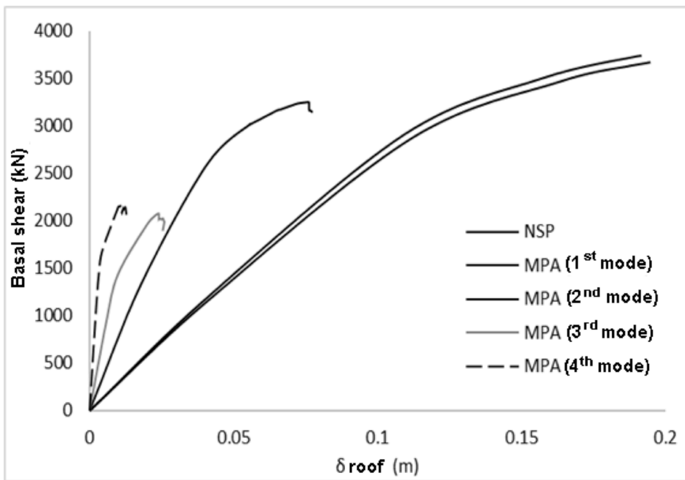

Figure 11. *Pushover analysis of direction X*

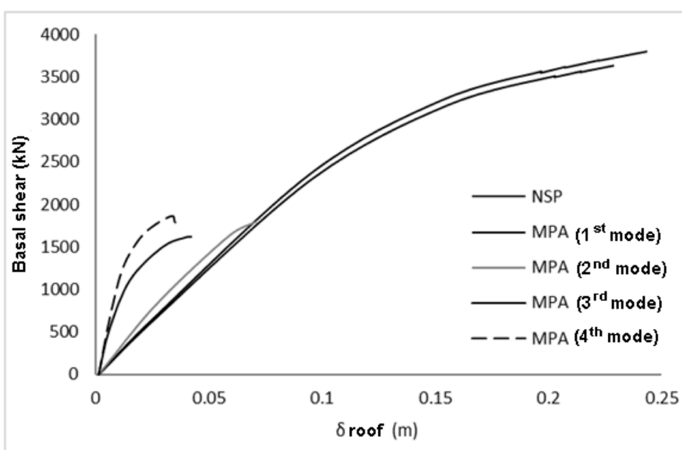

Figure 12. *Pushover analysis of direction Y*

3.2 Floor displacements

(Table 4) shows the results of displacements in direction X for the three types of analyses carried out.

Table 4. *Maximum displacements of the center of mass*

Method	*Max. Displ. X (m)*	*Max. Displ. Y (m)*
NSP	*0.1946*	*0.2283*
MPA	*0.2083*	*0.2591*
NLRHA	*0.2674*	*0.2219*

It can be observed that in direction X the displacements obtained through MPA are similar to those obtained through NSP, with an error of only -5.57%.

Although the MPA results are the closest to the NLRHA, the error reaches -22.87%, which shows the deficiency of MPA and NSP applied to torsionally-flexible structures. (Figure 13) shows the difference of displacements in all the floors.

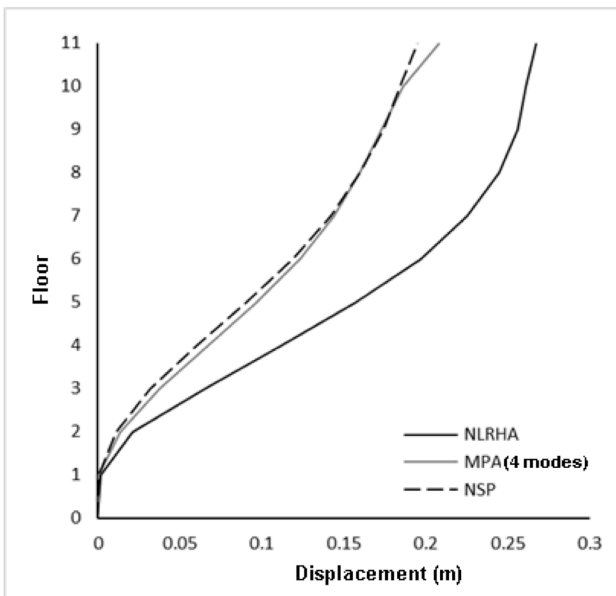

Figure 13. *Displacements in the center of mass of direction X*

On the other hand, in direction Y of the structure, the most adjusted displacements to the NLRHA are obtained through NSP, with an error of only 2.9%. It is important to mention that in this sense of analysis, the structure presents greater irregularity. For this reason, the MPA data tend to overestimate the displacement capacity of the structure. (Figure 14) shows the displacements for each floor for direction Y.

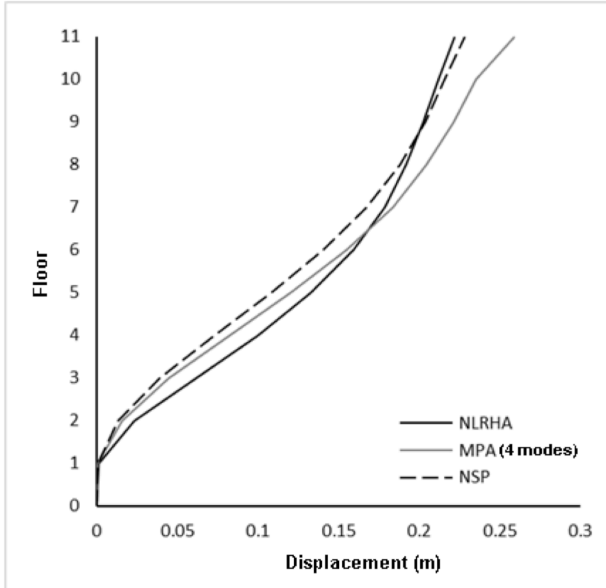

Figure 14. *Displacements in the center of mass of direction Y*

3.3 Floor drifts

The differences between the three types of analyses are more noticeable in the floor drifts. (Figure 15) shows that in direction X, as in the displacements, the drifts in all the floors are much greater than those obtained through NSP and MPA, being the NSP the most approximate. The error in the maximum drift reaches -36.51%.

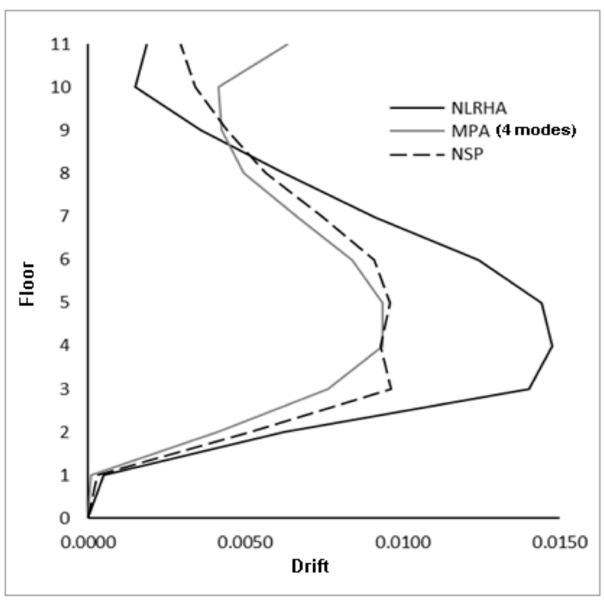

Figure 15. *Drifts in the center of mass of direction X*

In direction Y, the highest displacement demand was obtained through MPA. However, the maximum drift (floor 3) is obtained through NLRHA. This indicates that not always a greater displacement results in a greater drift (Figure 16).

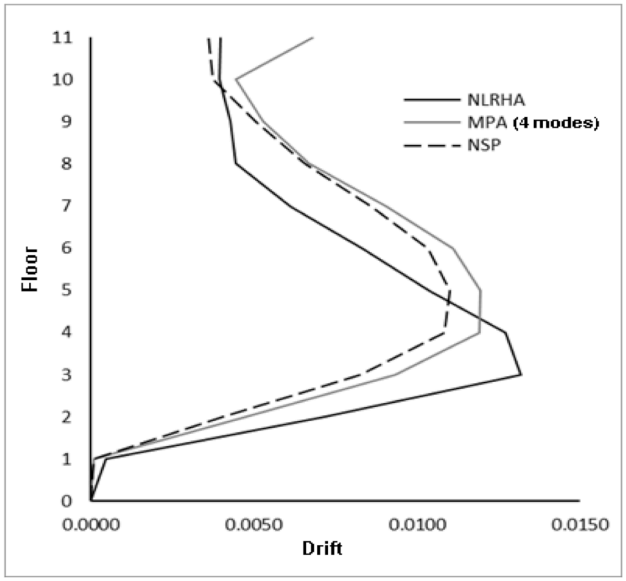

Figure 16. Drifts in the center of mass of direction Y

3.4 Number of Modes for MPA

In the MPA, the number of modes considered significantly influences the response since the results are closer to the NLRHA as the higher amount of modal mass is excited.

When considering one mode for direction X, only 59.2% of the modal mass is excited, so the results tend to be inaccurate. In addition, Figure 17 shows the influence of higher vibration modes on the response, especially on the top floors.

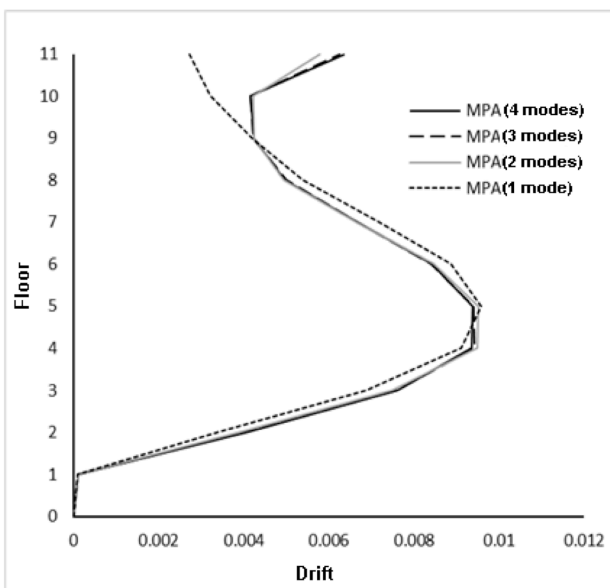

Figure 17. Floor drifts estimated through MPA including 1, 2, 3 and 4 modes. Direction X

Similar results are obtained in direction Y, where the first mode accumulates 44% of the modal mass. This requires using more vibration modes to achieve greater accuracy (Figure 18).

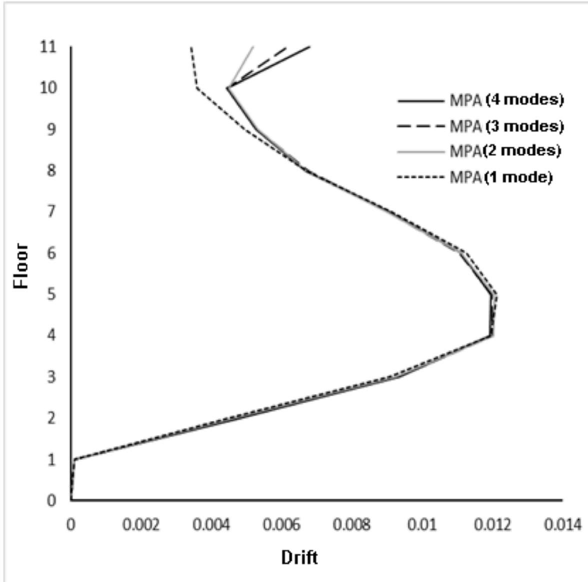

Figure 18. Floor drifts estimated through MPA including 1, 2, 3 and 4 modes. Direction Y

3.5 Failure Mechanism

Concerning the failure mechanism, (Figure 19) shows that through NLRHA, the damage is concentrated in the intermediate floors of the structure (3rd and 4th floor), which governs the collapse of the structure.

However, there is also considerable damage to the top floor columns, which exceed the collapse prevention level. This proves the effect of higher vibration modes on structures with T > 1 sec.

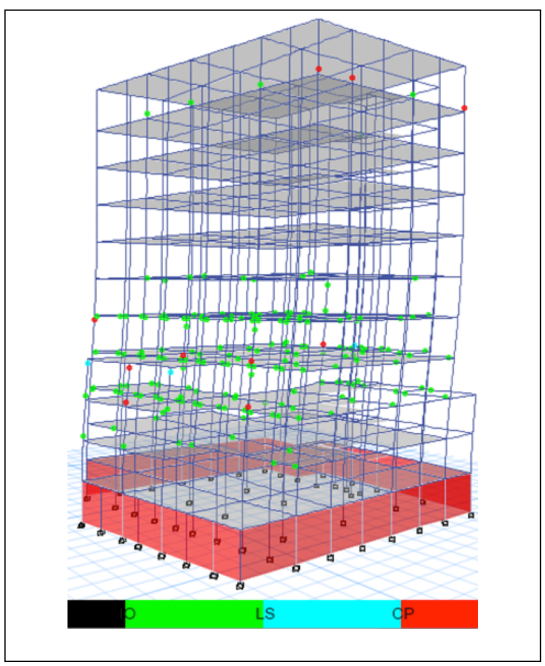

Figure 19. Location of plastic hinges determined through NLRHA for the Loma Prieta earthquake

The MPA shows that for the fundamental mode, failure is concentrated in the intermediate floors as well as in the NLRHA, while the upper floors remain elastic. Therefore, for these floors to contribute to the response, it is necessary to consider modes 2, 3 and 4, in which the failure is concentrated on the upper floors. (Figure 20) shows the results.

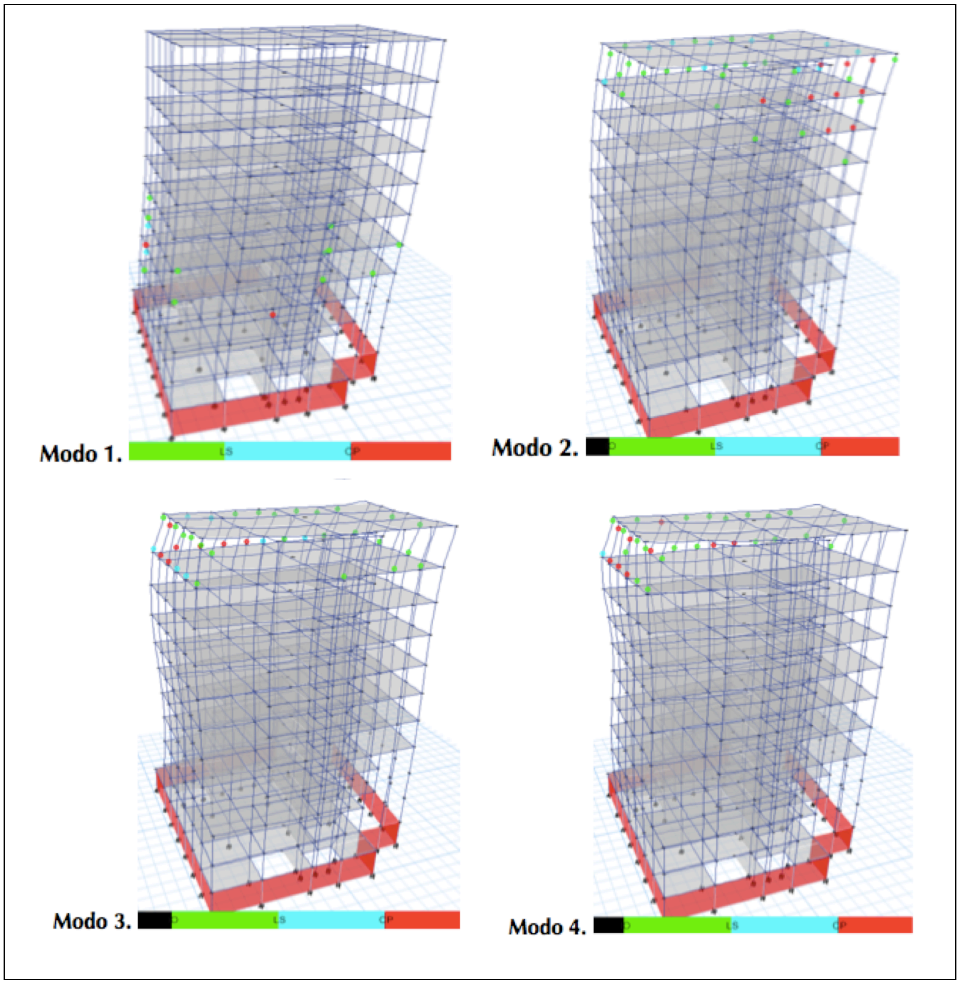

Figure 20. Location of plastic hinges determined through MPA for the 1st, 2nd, 3rd, and 4th modes

Finally, through NSP, the damage is only concentrated in the intermediate floors, with the rest of the floors remaining elastic, so the results are very different from those reported by the NLRHA (Figure 21).

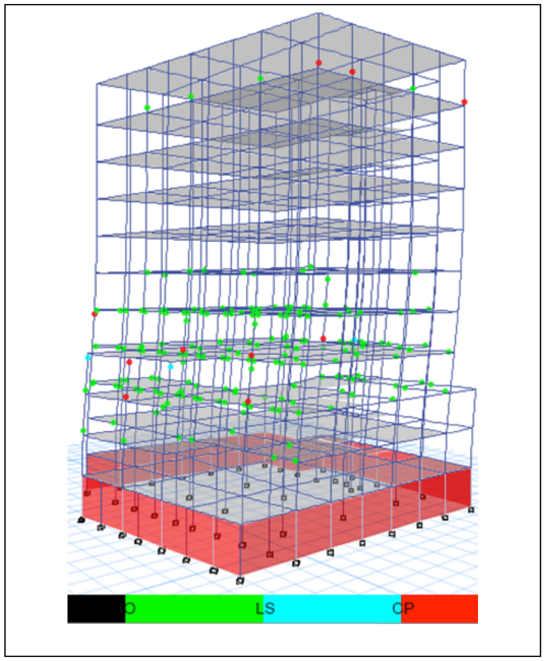

Figure 21. Location of plastic hinges determined through NSP

4. Conclusions

It could be verified that the displacement results obtained through NSP do not entirely reflect the response in displacements of unsymmetric structures, with a modal mass excited in the first vibration mode of less than 75%.

Similarly, although MPA applies to structures whose higher vibration modes considerably influence the displacement response, it is not applicable when the structure is torsionally flexible, which is characteristic of unsymmetric structures. However, this methodology shows fewer errors than NSP in comparison with NLRHA.

For direction Y, the maximum displacement obtained through MPA is greater than the one obtained through NLRHA. This occurs because the structure presents greater irregularities in this direction since the percentage of modal mass excited in torsion reaches 24.1% in the fundamental mode. In this context, the results of MPA displacements tend to magnify, showing a false capacity of the structure.

It could be verified that although the distribution of drifts in height obtained by the three methods is similar (Figures 14) y (Figure 15), both NSP and MPA underestimate the maximum inelastic drifts. Although in one of the directions, the MPA displacement is greater than NLRHA, this does not guarantee that the drift will be greater.

The number of modes influences the error when determining the answer through MPA. It was determined that to make the response as close as possible to the NLRHA, the four translational modes should be considered in the analyzed direction.

Besides, when analyzing the failure mechanism, it was established that the results obtained from the combined response of the MPA, including the four vibration modes, are very similar to the results obtained through NLRHA. In the first mode, the failure is concentrated on intermediate floors, while for higher modes, the failure is generated on the upper floors.

Also, the failure mechanism obtained through NSP does not reflect the contribution of the upper floors in the response since these remain elastic.

In conclusion, in the case of NSP, the results in the displacements and in the failure mechanism can be inaccurate because these structures have T > 1sec, and in the case of MPA, the response to displacements is not accurate because these structures are unsymmetrical, but the results are accurate in terms of the failure mechanism. For this reason, in these cases, the results must be confirmed through NLRHA.

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